G E O M E T R Y

CHAPTER 6

QUADRILATERALS

Notes & Study Guide

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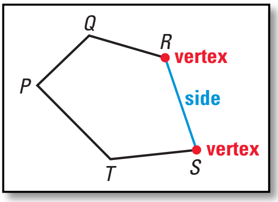
INTRODUCTION

In Chapter 6, we move from triangles to shapes with more sides, specifically four. We will look at the five main 4-sided shapes and their characteristics.

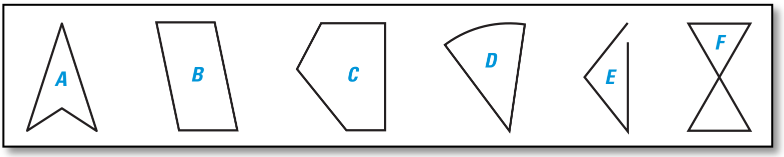
→ POLYGONS

Any **closed**, **plane** figure that is built from three or more segments (no curves!) is called a **polygon**.



"poly" means many and "-gon" means sides.

Closed *– no gaps or "crossovers" in the segments*
Plane *– flat or two-dimensional*



A, B & C are all polygons. D, E & F are not polygons.

→ CLASSIFYING (NAMING) POLYGONS

Polygons get their names from how many sides they have. In general, we put the correct prefix in front of the "-gon" suffix. There are some exceptions.

Sides	Name	Sides	Name
3	**TRI**ANGLE	8	**OCTA**GON
4	**QUADRI**LATERAL	9	**NONA**GON
5	**PENTA**GON	10	**DECA**GON
6	**HEXA**GON	12	**DODECA**GON
7	**SEPTA**GON or **HEPTA**GON	n	**n–**gon

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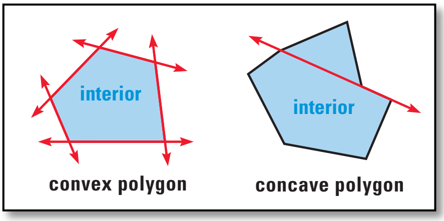
→ CONCAVE v. CONVEX

CONCAVE polygons are "dented". That means if one side was extended, it would go through the interior.

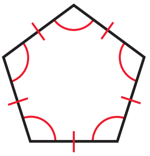
CONVEX polygons are not dented. None of the sides, when extended, will enter the interior.



→ REGULAR POLYGONS

When a polygon has all sides congruent (equilateral) and all angles congruent (equiangular) at the same time, it is referred to as **regular**.

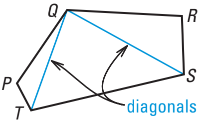
All regular polygons must be drawn in the nice "circular" way. As usual, look for lots of tick and angle marks.



→ DIAGONALS

A **diagonal** of a polygon is a segment that joins any two *nonconsecutive* vertices.

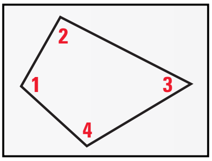
Non-consecutive means NOT next to each other.



→ INTERIOR ANGLES OF QUADRILATERALS

A **quadrilateral** is a polygon with four sides. "quad-" means four and "-lateral" means sides. A pattern exists with the interior angles of all quadrilaterals.

THEOREM 6.1 – The sum of the interior angles of any quadrilateral will ALWAYS be **360°**.



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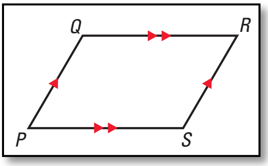
INTRODUCTION

Polygons that have four sides are called quadrilaterals. There are ***six*** *different quadrilaterals. Some are unique, but most of them share features. In this chapter we will study each of these six shapes.*

→ PARALLELOGRAMS

A quadrilateral (4-sided shape) that has **both pairs** of opposite sides parallel is called a **parallelogram**.

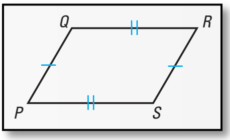
Shown: parallelogram PQRS or $\square PQRS$



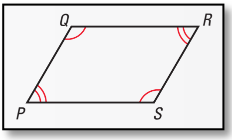
→ PROPERTIES OF PARALLELOGRAMS

Besides having four sides and 2 pairs of parallel lines, parallelograms have many other important properties. Each one is represented by its own theorem.

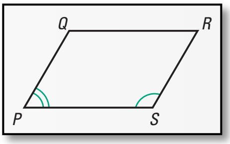
THEOREM 6.2 – If a quadrilateral is a parallelogram, then both pairs of **opposite sides are congruent**.



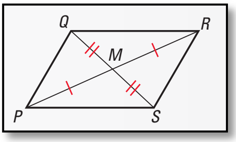
THEOREM 6.3 – If a quadrilateral is a parallelogram, then both pairs of **opposite angles are congruent.**



THEOREM 6.4 – If a quadrilateral is a parallelogram, then its' **consecutive angles are supplementary**.



THEOREM 6.5 – If a quadrilateral is a parallelogram, then its' **diagonals will bisect each other**.



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→ PROVING QUADRILATERALS ARE PARALLELOGRAMS

To prove that a quadrilateral is a parallelogram, you need to do the following…

(1) establish that the shape has four sides *(quadrilateral)*
(2) establish that the shape meets any one of the four properties

Each possible way to prove this is represented by its' own theorem
(Theorems 6.6 – 6.9)
SEE TEXTBOOK PAGE 340

These theorems are nothing more than the converses of the ones listed earlier in the notes.

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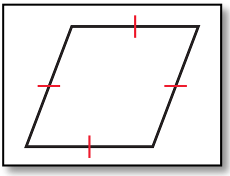
By definition, all we need to have for a parallelogram is both pairs of opposite sides parallel.

BUT, if we add something extra to that, then the parallelogram can turn into one of three **special parallelograms** (which are also quadrilaterals).

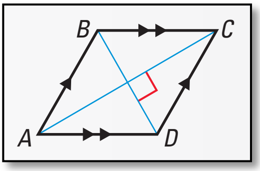
→ RHOMBUS

If a parallelogram has four congruent sides, then it is called a **rhombus**.

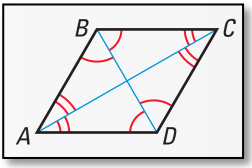


PROPERTIES OF RHOMBUSES

THEOREM 6.11 – A parallelogram is a rhombus IFF its **diagonals are perpendicular**.



THEOREM 6.12 – A parallelogram is a rhombus IFF each **diagonal bisects a pair of opposite angles**.



Notice that each of these Theorems say what a parallelogram must have in order to be called a rhombus.

This means that you must meet the requirements of the parallelogram first ***before*** *you can try to classify it as a rhombus.*

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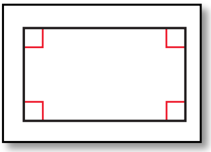
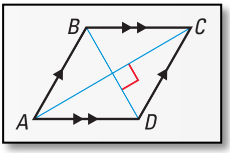
→ RECTANGLE

If a parallelogram has **four right angles**, then it is called a **rectangle**.



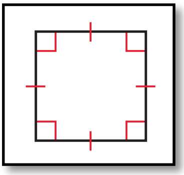
PROPERTIES OF RECTANGLES

THEOREM 6.13 – A parallelogram is a rectangle IFF its **diagonals are congruent**.



→ SQUARE

If a parallelogram has **four congruent sides AND four right angles**, then it is called a **square**.



PROPERTIES OF SQUARES

Notice that squares require properties of rhombuses AND rectangles…
(four congruent sides….four right angles)

Therefore, the properties of square are the same as those for rhombuses and rectangles.

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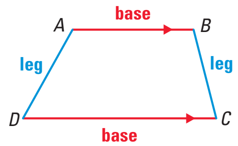
There are some quadrilaterals that do not meet the requirements of being parallelograms. These two shapes are the last two that we will examine.

→ TRAPEZOID

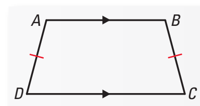
A quadrilateral that has exactly one pair of parallel sides is called a **trapezoid**.



The two sides that are parallel are called the **bases**.
The two sides that are not parallel are called the **legs**.

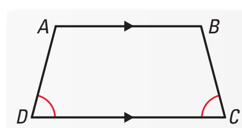
The angles that are on each end of the bases are called **base angles.**

If the two legs are congruent, then the trapezoid is called an **isosceles trapezoid**.



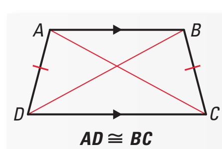
→ PROPERTIES OF TRAPEZOIDS

THEOREM 6.14 – If a trapezoid is isosceles, then each pair of base angles is congruent



THEOREM 6.15 – If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

THEOREM 6.16 – A trapezoid is isosceles IFF its' diagonals are congruent.



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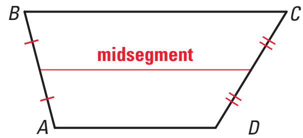
In a triangle, a midsegment was a segment that connected the midpoints of any two of its' sides.

For a trapezoid, the midsegment is a swegment that connects the **midpoints of the two legs**.



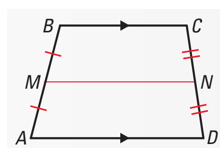
PROPERTIES OF TRAPEZOID MIDSEGMENTS

Trapezoid midsegments have two main properties that are similar to the ones for triangles. They are represented by a Midsegment Theorem.

The midsegment of a trapezoid…

- → …is parallel to each of the two bases AND
- → …is half the sum of the two bases

THEOREM 6.17 – The midsegment of a trapezoid is parallel to the bases AND is half the sum of the lengths of the bases.



Shown: MN is parallel to BC and AD. Also, MN = ½(BC + AD)

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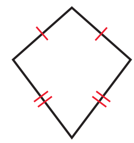
The last quadrilateral we will look at is easy to recognize and work with, since it shares very few properties with the others.

→ KITE

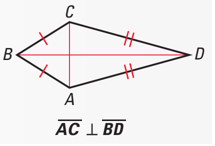
A quadrilateral that has two pairs of consecutive sides congruent is called a **kite**.



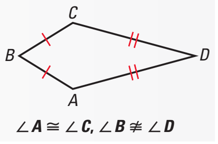
NOTE! In a kite, opposite sides are NOT congruent!

PROPERTIES OF KITES

THEOREM 6.18 – If a quadrilateral is a kite, then its diagonals are perpendicular.



THEOREM 6.19 – If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent. (across from the shorter diagonal)



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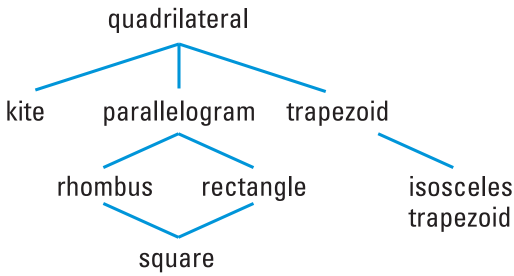
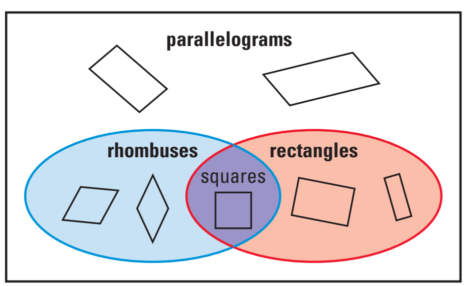
The diagram shown here shows how the six quadrilaterals are organized.



The figure below (called a Venn Diagram) shows how the special parallelograms are categorized.



Notice how a square is a type of rhombus or rectangle. Also notice how rhombuses, rectangles and squares are special types of parallelograms

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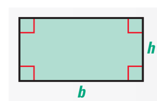
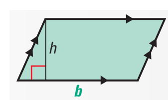
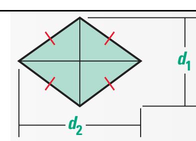
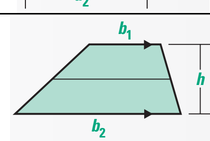
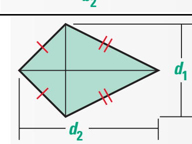
In geometry we are often interested in the numeric measurements of shapes such as perimeter, area, surface area and volume.

→ IMPORTANT AREA CONCEPTS

Area Congruence Postulate – *If two polygons are congruent, then their areas are the same.*

Area Addition Postulate – *If a figure is made from several smaller figures, then the area of the whole thing is equal to the sum of the individual pieces.*

QUADRILATERAL AREA FORMULAS

Rectangle	$A = b \cdot h$ $A = L \cdot W$	
Square	$A = s^2$	
Parallelogram	$A = b \cdot h$	
Rhombus	$A = \frac{1}{2}(d_1)(d_2)$	
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$	
Kite	$A = \frac{1}{2}(d1)(d2)$	

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