# Geometry Notes

Name ___Key___

8.1 Find Angle Measures in Polygons

Remember, the term **POLYGON** means "many-angled". For a figure to be classified as a polygon it must be a closed figure formed by segments in a plane so that:
- The sides that have a common endpoint are <u>not</u> collinear
- Each side intersects exactly two other sides <u>at their endpoints only</u>

Convex Polygon: If you extend any segment to make a line, no point on the line is on the interior of the polygon. **Concave Polygon:** A polygon that is not convex.

A convex polygon is called a **regular polygon** if all the sides are the same length and all the angles are congruent.

_Convex_   _Concave_   _Regular, Convex_
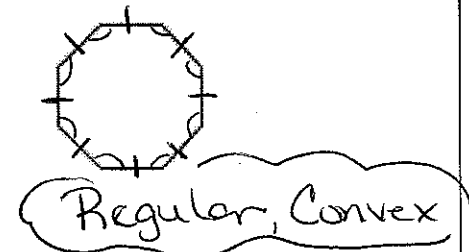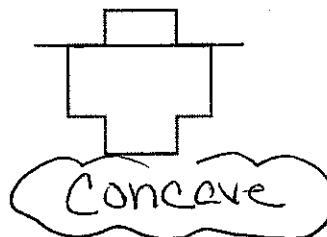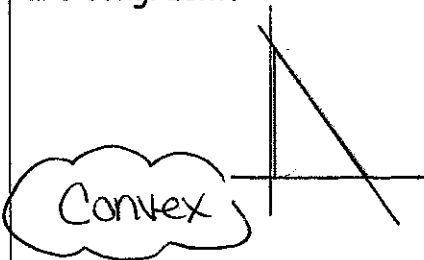
Diagonal of a polygon:

a segment that joins 2 non consecutive vertices
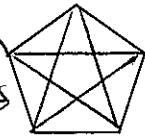
In each of the convex polygons below, pick one vertex and draw all the possible diagonals only from that one vertex. Do this for each polygon and fill in the table with the number of sides and the number of triangles formed.

If the sum of the angles in one triangle is 180°, what is the sum of the interior angles in each polygon? Fill in the table for each polygon.
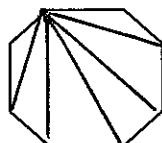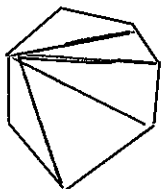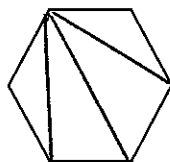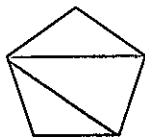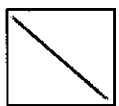
# of Sides	# of triangles	Sum of int. angles
3	1	180°
4	2	360°
5	3	540°
6	4	720°
7	5	900°
8	6	1080°
n	n−2	(n−2)180°

Sum of Interior Angles

If s is the **sum of the measures of the interior angles** of a convex n-gon, then

$$\text{Sum of int. } \angle s = (n-2)180°$$

$$S = (n-2)180°$$

Examples

1. Find the sum of the interior angles of a regular, convex 82-gon. $$Sum = (82-2)\,180°$$ $$Sum = (80)\,180°$$ $$\boxed{Sum = 14,400°}$$	2. The sum of the measures of the interior angles of a convex polygon is 2520. Classify the polygon by the number of sides. $$Sum = (n-2)\,180°$$ $$2520 = (n-2)\,180°$$ $$14 = n-2$$ $$16 = n$$ $$\boxed{\# \text{ of sides} = 16 \quad 16\text{-gon}}$$

Exterior Angle Measure

Exterior angles are formed by extending the side of a polygon in one direction. Each of the angles formed on the exterior of the pentagon below are exterior angles. Assume this is a regular polygon.
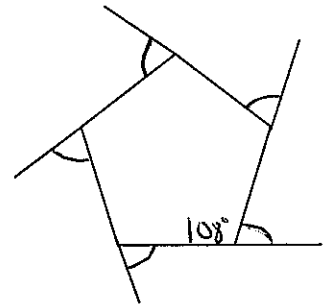
1. What is the sum of the interior angles? $(5-2)180° = \boxed{540°}$

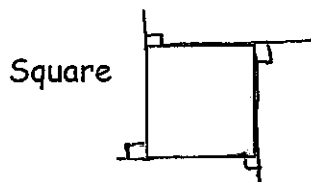2. What do we know about the interior angles? $\boxed{\text{they are } \cong}$

3. What is the measure of each interior angle? $540 \div 5 = \boxed{108°}$

4. What is the measure of each exterior angle? $180 - 108 = \boxed{72°}$

5. What is the sum of the measures of the exterior angles? $5(72°) = \boxed{360°}$
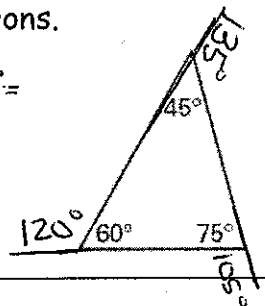

Find the **sum** of the measures of the exterior angles of the following polygons.

Square $4(90) = \boxed{360°}$

$135° + 120° + 105° = \boxed{360°}$

135° 45° 120°/60° 75° 105°

Exterior Angle Sum Theorem:

In a convex polygon, the sum of the measures of the exterior angles (one at each vertex) **is:**

$$\boxed{360°}$$

The measure of an exterior angle of a regular polygon is given. Find the number of sides.

1. 30°	2. 20°	3. 5°
$\dfrac{360°}{30°} = 12$	$\dfrac{360°}{20°} = 18$	$\dfrac{360°}{5°} = 72$
12 sides	18 sides	72 sides
dodecagon	18-gon	72-gon

4. The measures of the **interior angles** of a pentagon are 2x, 2x, x+45, x-3, and 4x-132.
    Find the measure of each interior angle. Is the pentagon a **regular polygon?**
    What is the **sum of the measures of its exterior angles?**

5 sides
(5-2)180
(3)(180)
$\boxed{540°}$

$2x + 2x + x + 45 + x - 3 + 4x - 132 = 540$
$10x - 90 = 540$
$10x = 630$
$\boxed{x = 63}$

126°, 126°, 108°, 60°, 120°
not a regular poly
Sum of ext ∠s = 360°

5. The measure of the interior angle of a regular polygon is 157.5°.
   Find the **number of sides** of the polygon. What is the measure of **one exterior angle?**

157.5 / 22.5

one ext ∠ = 22.5°

$\dfrac{360°}{22.5°} = 16$

16 sides

16-gon

6. Find the measures of an interior and an exterior angle of a regular, convex 40-gon.

(n-2) 180 = int sum
(40-2)180
(38)180
6840 = sum

$\dfrac{6840}{40} = 171°$

180 - 171 = 9°

one int ∠ = 171°
one ext ∠ = 9°