GOAL Volumes of pyramids and cones.

Vocabulary

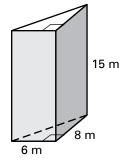
Theorem 9 Volume of a Pyramid: The volume V of a pyramid is $V = \frac{1}{3}Bh$ where B is the area of the base and h is the height.

Theorem 10 Volume of a Cone: The volume V of a cone is $V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2h$, where B is the area of the base, h is the height, and r is the radius of the base.

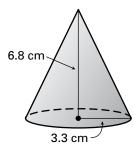
EXAMPLE 1 Find the volume of a solid

Find the volume of a solid.





b.



Solution

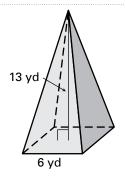
a.
$$V = \frac{1}{3}Bh = \frac{1}{3}(\frac{1}{2} \cdot 6 \cdot 8)(15) = 120 \text{ m}^3$$

b.
$$V = \frac{1}{3}Bh = \frac{1}{3}(\pi r^2)(h) = \frac{1}{3}(\pi \cdot 3.3^2)(6.8)$$

$$V = 24.684\pi \approx 77.55 \text{ cm}^3$$

Exercise for Example 1

1. Find the volume of the pyramid. Round your answer to two decimal places.



EXAMPLE 2

Use volume of a pyramid

The pyramid has a height of 177 meters and volume of 3,465,825 cubic meters. Find the side length of the square base.



Solution

$$V = \frac{1}{3}Bh$$

Write formula.

$$3,465,825 = \frac{1}{3}(x^2)(177)$$

Substitute.

$$10,397,475 = 177x^2$$

Multiply each side by 3.

$$58,743 \approx x^2$$

Divide each side by 177.

$$242 \approx x$$

Find the positive square root.

The side length of the base is about 242 meters.

EXAMPLE 3

Use trigonometry to find the volume of a cone

Find the volume of the right cone.

Solution

To find the radius *r* of the base use trigonometry.

$$\tan 59^{\circ} = \frac{\text{opp.}}{\text{adj.}}$$

Write ratio.

$$\tan 59^\circ = \frac{20}{r}$$

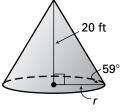
Substitute.

$$r = \frac{20}{\tan 59^{\circ}} \approx 12.02$$

Solve for r.

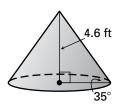
Use the formula for the volume of a cone.

$$V = \frac{1}{3}Bh = \frac{1}{3}(\pi r^2)(h) = \frac{1}{3}\pi(12.02^2)(20) \approx 3025.99 \text{ ft}^3$$



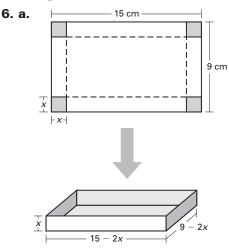
Exercises for Examples 2 and 3

- 2. The volume of a right cone is 1275π cubic meters and the radius is 15 meters. Find the height of the cone. Round your answer to two decimal places.
- **3.** Find the volume of the cone at the right. Round your answer to two decimal places.



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Lesson 11.6 Volume of Prisms and Cylinders, continued



b.
$$V = x(9 - 2x)(15 - 2x) = 4x^3 - 48x + 135$$

C.

x	0.8	1.2	1.8	2.0	2.2	2.6
V	79.3	99.8	110.8	110	107.3	96.8

Length: 11.4 cm; width: 5.4 cm; height: 1.8 cm

Lesson 11.7 Volume of Pyramids and Cones

Teaching Guide

1. 9 **2.** about 848.2 ft³ **3.** about 31.4 yd³

4. The contractor only needs 25 cubic yards and has 31.4 cubic yards available. **5.** No; There is about 22 cubic yards of gravel, so there is not enough.

Practice Level A

1. $9\sqrt{3} \approx 15.6$ square units **2.** 64 square units

3. $16\pi \approx 50.3$ square units **4.** 400 yd^3 **5.** 20 m^3

6. 65.33 in.³ **7.** 336 cm³ **8.** 86.6 ft³

9. 249.42 cm^3 **10.** 100.53 in.^3 **11.** 287.98 cm^3

12. 75.40 mm³ **13.** 117.29 yd³ **14.** 314.16 ft³

15. 25.13 m³ **16.** x = 10 cm **17.** x = 5 ft

18. x = 11 m **19.** 8 ft **20.** 1005.31 m³

21. 17.34 ft³ **22.** 638.98 cm³ **23.** 323.04 yd³

24. 79.52 cm³ **25.** about 6.77 ft

Practice Level B

1. 100.53 cm^3 **2.** 20 in.^3 **3.** 10.67 cm^3

4. 414.69 m^3 **5.** 126 in.^3 **6.** 163.49 cm^3

7. 6 in. **8.** 7 cm **9.** 7 m **10.** C

11. 2035.75 cm³ **12.** 3681.88 m³ **13.** 2652.53 ft³

14. 448 m³ **15.** 90.93 in.³ **16.** 144 cm³

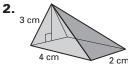
17. 190.87 mm³ **18.** 103.67 in.³

19. 122.67 cm^3 **20.** 12 ft **21.** 1520.53 ft^3

22. 56.32 yd³ **23.** no

Practice Level C

1. 3 in.



28.27 in.³

 8 cm^3

3. 169.76 m^3 **4.** 6.58 ft^3 **5.** 124.05 cm^3

6. 14.76 yd^3 **7.** $4,579,109.32 \text{ m}^3$ **8.** 50.20 in.^3

9. 2211.8 cm³ **10.** 29,605.40 mm³

11. 353.97 m^3 **12.** 10 yd **13.** 502.81 m^3

14. 181.83 in.³ **15.** 178.63 cm³ **16.** 963.4 cm³

17. 24 in.³ **18.** No, each cone would require about 5.06 grams of gold. For all twelve, the jeweler would need about 60.7 grams.

19. 1840 in.³ **20.** $533\frac{1}{3}$ m³

Study Guide

1. $V = 156 \text{ yd}^3$ **2.** h = 17 m **3.** $V = 207.9 \text{ in.}^3$

Problem Solving Workshop: Worked Out Example

1. 11.5 in. **2.** 1.61 in. **3.** 42.41 in.²

Challenge Practice

1. $\frac{7\sqrt{3}}{2} \approx 6.1$ in. **2.** Cone; $100\pi \approx 314.2$ cm³

3. Frustum of a cone; $\frac{5056\pi}{3} \approx 5294.6 \text{ ft}^3$

4. $V = \frac{1}{3}b^2H - \frac{1}{3}a^2(H-h)$

5. $\frac{H}{b} = \frac{H-h}{a}$; $H = \frac{bh}{b-a}$; $H-h = \frac{ah}{b-a}$

6. $V = \frac{1}{3}h(a^2 + ab + b^2)$ **7.** 912 m³

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