GOAL

Find volumes of prisms and cylinders.

Vocabulary

The **volume** of a solid is the number of cubic units contained in its interior.

Postulate 27 Volume of a Cube Postulate: The volume of a cube is the cube of the length of its side.

Postulate 28 Volume Congruence Postulate: If two polyhedra are congruent, then they have the same volume.

Postulate 29 Volume Addition Postulate: The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

Theorem 6 Volume of a Prism: The volume *V* of a prism is V = Bh where B is the area of a base and h is the height.

Theorem 7 Volume of a Cylinder: The volume *V* of a cylinder is $V = Bh = \pi r^2 h$, where B is the area of a base, h is the height, and r is the radius of a base.

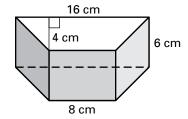
Theorem 8 Cavalieri's Principle: If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

EXAMPLE 1

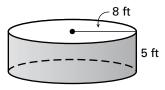
Find volumes of prisms and cylinders

Find the volume of the solid.

Right trapezoidal prism



b. Right cylinder



Solution

- **a.** The area of the base is $\frac{1}{2}(4)(16 + 8) = 48 \text{ cm}^2$ and h = 6 cm. $V = Bh = 48(6) = 288 \text{ cm}^3$
- **b.** The area of the base is $\pi r^2 = (8)^2 = 64\pi$ ft². Use h = 5 ft to find the volume.

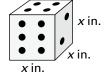
$$V = Bh = 64\pi(5) = 320\pi \approx 1005.31 \text{ ft}^3$$

Study Guide continued

For use with the lesson "Volume of Prisms and Cylinders"

EXAMPLE 2 Use volume of a prism

The volume of the cube is 135 cubic inches. Find the value of x.



Solution

A side length of the cube is *x* inches.

 $V = x^3$ Formula for volume of a cube

 $135 = x^3$ Substitute for V.

 $5.13 \approx x$ Find the cube root.

The value of x is about 5.13 inches.

Exercises for Examples 1 and 2

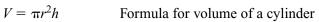
- **1.** Find the volume of a square prism that has a base edge length of 6 feet and a height of 13 feet.
- 2. The volume of a right cylinder is 896π cubic inches and the height is 14 inches. Find the radius.

EXAMPLES Find the volume of an oblique cylinder

Find the volume of the oblique cylinder.

Solution

Cavalieri's Principle allows you to use Theorem 7 to find the volume of the oblique cylinder.



$$V = \pi(7)^2(10)$$
 Substitute for known values.

 $V = 490\pi$ Simplify.

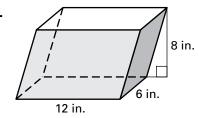
$$V \approx 1539.38$$
 Use a calculator.

The volume of the oblique cylinder is about 1539.38 cubic centimeters.

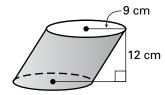
Exercises for Example 3

Find the volume of the oblique prism or cylinder shown.

3.



4.



7 cm

10 cm

Lesson 11.6 Volume of Prisms and Cylinders, continued

Investigating Geometry Activity

Explore

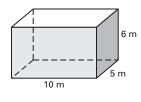
	Length of Base	Width of Base	Height of Prism
Prism 1	6 cubes	3 cubes	2 cubes
Prism 2	3 cubes	2 cubes	4 cubes
Prism 3	5 cubes	3 cubes	2 cubes

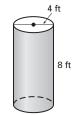
	Number of Cubes in Base	Number of Cubes in Prism
Prism 1	18 cubes	36 cubes
Prism 2	6 cubes	24 cubes
Prism 3	15 cubes	30 cubes

- **1.** Number of Cubes in Prism **2.** The volume is the product of the dimensions. **3.** $V = \ell wh$
- **4.** It is the product of the length and width of the base. It represents the area of the base.
- **5.** Sample answer: V = Bh, where B represents the area of the base.

Practice Level A

- **1.** 40 cubic units **2.** 240 cubic units
- **3.** 84 cubic units **4.** 8 ft³ **5.** 105 m^3 **6.** 40 cm^3
- **7.** 346.41 yd^3 **8.** 210 in.^3 **9.** 168 m^3
- **10.** 452.39 m³ **11.** 307.88 cm³ **12.** 6082.12 ft³
- **13.** 593.76 yd³ **14.** 1990.98 in.³
- **15.** 1082.12 mm³ **16.** 3.91 yd **17.** 15 cm
- **18.** 300 m³
- **19.** 100.53 ft³



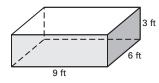


- **20.** 62.83 in.^3 **21.** 800 cm^3 **22.** 4000 ft^3
- **23.** 5184 ft³ **24.** 12,723.45 ft³

Practice Level B

- **1.** 120 cubic units **2.** 120 cubic units
- **3.** 136 cubic units **4.** 96 m 3 **5.** 189 in. 3
- **6.** 831.38 cm^3 **7.** 113.1 ft^3 **8.** 942.48 in.^3

- **9.** 276.46 cm³ **10.** 12 m **11.** 6 ft
- **12.** 6 cm **13.** C
- **14.** 162 ft³
- **15.** 402.12 m³

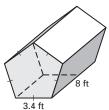




- **16.** 48 mm³ **17.** 502.65 in.³ **18.** 72 mm³
- **19.** 62.83 in.^3 **20.** 301.59 cm^3 **21.** 10.39 in.^2
- **22.** 124.71 in.³ **23.** no

Practice Level C

- **1.** 332.55 m^3 **2.** 365.88 ft^3 **3.** $122,850 \text{ yd}^3$
- **4.** 1.97 in.³ **5.** 2209.55 cm³ **6.** 1005.46 m³
- **7.** 45.1 in.³ **8.** 996.83 m³ **9.** 297,362.79 yd³
- **10.** 8293.8 cm³ **11.** 3443.1 ft³ **12.** 589.35 in.³
- **13.** 3.5 mm **14.** about 7.4 yd **15.** about 5.05 cm
- **16.** 4.5 cm **17.** 12.4 in.
- **18.** 159.12 ft³
- **19.** 6456.5 m³





- **20.** 459.46 m³ **21.** 156.44 ft³ **22.** 393.3 yd³
- **23.** 2818.94 cm³ **24.** 508.75 ft³ **25.** 48.78 ft³

Study Guide

- **1.** $V = 468 \text{ ft}^3$ **2.** r = 8 in. **3.** $V = 576 \text{ in.}^3$
- **4.** $V = 972\pi \approx 3053.6 \text{ cm}^3$

Real-Life Application

- **1.** 1.57 in.^3 **2.** 18.84 in.^3 **3.** 30 in.^3 , 24 in.^3 , 150 in.^3 , 18 in.^3 **4.** $4 \text{ in.} \times 3 \text{ in.} \times 2 \text{ in.}$
- **5.** Yes; it can fit 4 along the length and 3 along the width and it is exactly high enough.
- **6.** 3 in. \times 2 in. \times 5 in.; two layers of 6 with packing material

Challenge Practice

- **1.** $1280\sqrt{3} \approx 2217.0 \text{ cm}^3$
- **2.** $125\pi\sqrt{3} \approx 680.2 \text{ in.}^3$
- **3.** $2040\sqrt{3}\approx 3533.4~\text{m}^3$ **4.** $40\pi\approx 125.7~\text{in.}^3;$ $40\pi-64\approx 61.7~\text{in.}^3$
 - **5. a.** $736\pi \approx 2312.2 \text{ cm}^3$ **b.** 5 in. **c.** 5 ft