

LESSON
11.5**Study Guide**

For use with the lesson "Explore Solids"

GOAL Identify solids.**Vocabulary**

A **polyhedron** is a solid that is bounded by polygons, called **faces**, that enclose a single region of space.

An **edge** of a polyhedron is a line segment formed by the intersection of two faces.

A **vertex** of a polyhedron is a point where three or more edges meet.

The **bases** of a prism are congruent polygons in parallel planes. The **base** of a pyramid is a polygon.

Theorem 1 Euler's Theorem: The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.

A polyhedron is **regular** if all of its faces are congruent regular polygons.

A polyhedron is **convex** if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron.

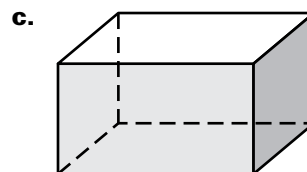
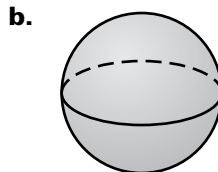
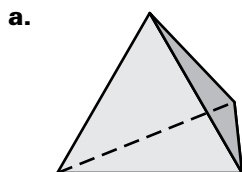
A polyhedron is *concave* if any two points on its surface can be connected by a segment that goes on the outside of the polyhedron.

Platonic solids are five regular polyhedra that include the regular tetrahedron, cube, regular octahedron, regular dodecahedron, and regular icosahedron.

A **cross section** is the intersection of a plane and a solid.

EXAMPLE 1 Identify and name polyhedra

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

**Solution**

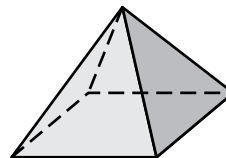
- The solid is formed by polygons, so it is a polyhedron. The base is a triangle, so it is a triangular pyramid. It has 4 faces, 4 vertices, and 6 edges.
- The sphere has a curved surface, so it is not a polyhedron.
- The solid is formed by polygons, so it is a polyhedron. The two bases are congruent rectangles, so it is a rectangular prism. It has 6 faces, 8 vertices, and 12 edges.

LESSON
11.5**Study Guide** *continued*
*For use with the lesson "Explore Solids"***EXAMPLE 2 Use Euler's Theorem with Platonic solids**

Find the number of faces, vertices, and edges of the polyhedron. Check your answer using Euler's Theorem.

Solution

By counting on the diagram, the tetrahedron has 5 faces, 5 vertices, and 8 edges. Use Euler's Theorem to check.



$$F + V = E + 2 \quad \text{Euler's Theorem}$$

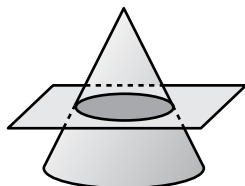
$$5 + 5 = 8 + 2 \quad \text{Substitute.}$$

$$10 = 10 \quad \text{This is a true statement. So, the solution checks.}$$

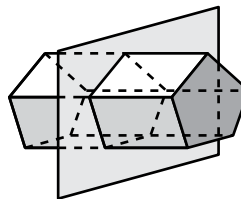
EXAMPLE 3 Describe cross sections

Describe the shape formed by the intersection of the plane and the solid.

a.



b.

**Solution**

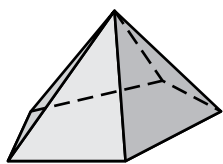
a. The cross section is a circle.

b. The cross section is a pentagon.

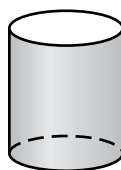
Exercises for Examples 1, 2, and 3

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

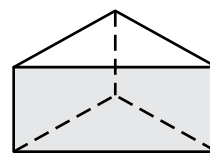
1.



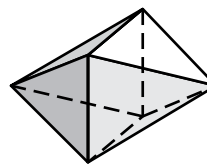
2.



3.

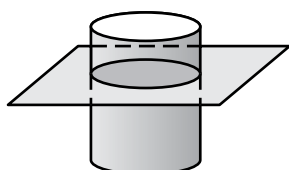


4. Find the number of faces, vertices, and edges of the polyhedron. Check your answer using Euler's Theorem.

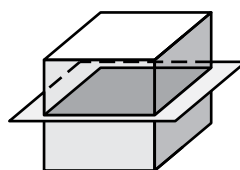


Describe the shape formed by the intersection of the plane and solid.

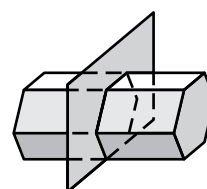
5.



6.



7.



Lesson 11.5 Explore Solids, continued

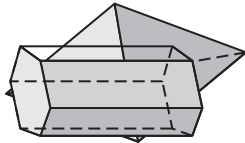
Practice Level B

1. yes; rectangular prism 2. no; The surfaces are not polygons. 3. no; The surfaces are not polygons.

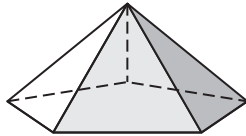
4. 4 5. 16 6. 36

7.

9.



8.



10. 5, 5, 8 11. 6, 8, 12 12. 5, 6, 9 13. 7, 7, 12

14. 8, 12, 18 15. 7, 10, 15 16. octagon

17. concave 18. convex 19. concave

20. pentagon 21. ellipse 22. rectangle 23. B

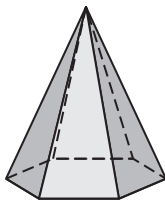
24. rectangular prism and rectangular pyramid; 9 faces 25. equilateral 26. $\sqrt{2}$ 27. 24

Practice Level C

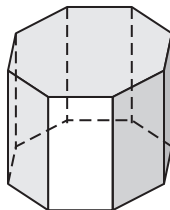
1. yes; heptagonal pyramid 2. no; Solid has a curved surface. 3. yes; nonagonal prism 4. 6

5. 36 6. 54

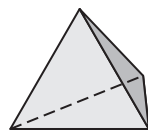
7.



8.



9.



10. 6 faces, 8 vertices, 12 edges 11. 10 faces, 16 vertices, 24 edges 12. 9 faces, 14 vertices, 21 edges 13. 12 faces, 20 vertices, 30 edges

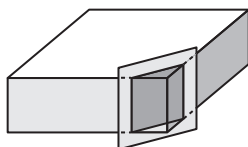
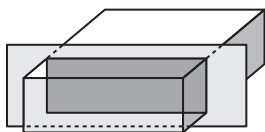
14. 12 faces, 20 vertices, 30 edges 15. 15 faces, 26 vertices, 39 edges 16. concave 17. concave

18. concave 19. rectangle 20. ellipse

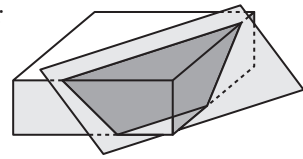
21. triangle 22. true 23. true 24. false

25. false

26. a. Sample answer: b. Sample answer:



c. Sample answer:



Study Guide

1. Yes; Pentagonal pyramid; 6 faces, 6 vertices, 10 edges 2. Not a polyhedron 3. Yes; Triangular prism; 5 faces, 6 vertices, 9 edges

4. 8 faces, 6 vertices, 12 edges; $F + V = E + 2$, $8 + 6 = 12 + 2$, so $14 = 14$ 5. The cross section is a circle. 6. The cross section is a square.

7. The cross section is a hexagon.

Interdisciplinary Application

1. No; a rhombus is not a regular polygon; all sides are congruent in a rhombus, but not all angles.

2. 24 3. No; unless the triangles are equilateral triangles, it cannot be a regular polyhedron.

4. 26 5. octagon

Challenge Practice

1. Faces: 4 triangles, 4 hexagons; vertices: 12; edges: 18 2. Faces: 8 triangles, 6 squares; vertices: 12; edges: 24 3. Faces: 8 triangles, 6 octagons; vertices: 24; edges: 36

4. Faces: 6 squares, 8 hexagons; vertices: 24; edges: 36 5. A rhombicuboctahedron has 26 faces and 24 vertices and a truncated cuboctahedron has 26 faces and 48 vertices.

6. An icosidodecahedron has 60 edges and 30 vertices and a truncated dodecahedron has 90 edges and 60 vertices.

Lesson 11.6 Volume of Prisms and Cylinders

Teaching Guide

1. square prism and a cylinder 2. square prism: 12 ft by 12 ft by 5 ft; cylinder: 6 ft and a height 5 ft 3. square prism: 720 ft^3 ; cylinder: about 565.5 ft^3

4. about 1285.5 ft^3 5. about 9615.44 gallons