Geometry Notes

Name Key

10.5 Apply Other Angle Relationships in Circles

Recall ....

You know that the measure of a **central angle** is equal to its intercepted arc.

You know that the measure of an **inscribed angle** is 1/2 of its intercepted arc.

ANGLES ON THE CIRCLE THEOREM

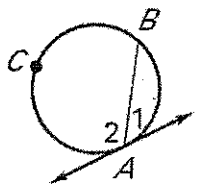
If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is **one half** the measure of its intercepted arc.

$\angle 1$ intercepts arc $\overset{\frown}{AB}$

$m\angle 1 = \frac{1}{2} m\overset{\frown}{AB}$

$\angle 2$ intercepts arc $\overset{\frown}{ACB}$

$m\angle 2 = \frac{1}{2} m\overset{\frown}{ACB}$

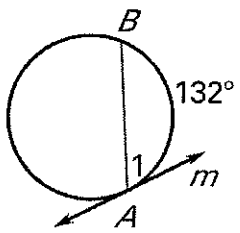


1. $m\angle 1 = 66°$

$\angle 1 = \frac{1}{2}\overset{\frown}{AB}$

$\angle 1 = \frac{1}{2}(132°)$

$\angle 1 = 66$

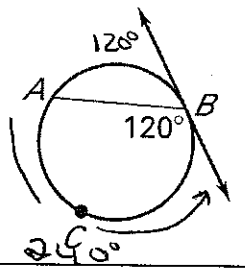


2. $m\overset{\frown}{AB} = 120°$

$120 = \frac{1}{2}\overset{\frown}{ACB}$

$240 = \overset{\frown}{ACB}$

$360 - 240 = \overset{\frown}{AB}$



ANGLES INSIDE THE CIRCLE THEOREM

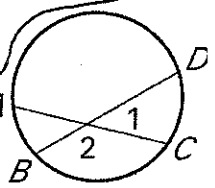
If two chords intersect ***inside*** **a circle**, then the measure of each angle is one half the ***sum*** of the measures of the arcs intercepted by the angle and its vertical angle.

$\angle 1$ intercepts arcs $\overset{\frown}{DC}$ and $\overset{\frown}{AB}$

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{DC} + m\overset{\frown}{AB})$

$\angle 2$ intercepts arcs $\overset{\frown}{BC}$ and $\overset{\frown}{AD}$

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{BC} + m\overset{\frown}{AD})$

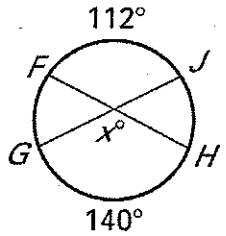


3. $x = 126$

$X = \frac{1}{2}(112 + 140)$

$X = \frac{1}{2}(252)$

$X = 126$



4. $x = 7$

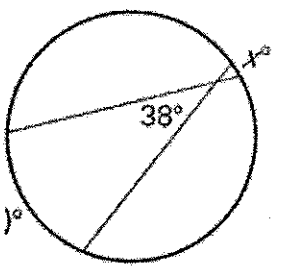
$38 = \frac{1}{2}(10x - 1 + x)$

$38 = \frac{1}{2}(11x - 1)$

$76 = 11x - 1$

$77 = 11x$

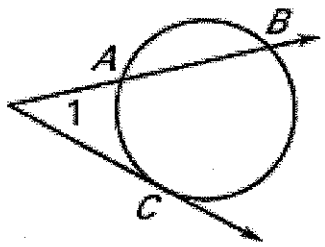
$7 = x$



ANGLES OUTSIDE THE CIRCLE THEOREM

If a tangent and a secant, two tangents, or two secants intersect ***outside*** **a circle**, then the measure of the angle formed is one half the ***difference*** of the measures of the intercepted arcs.

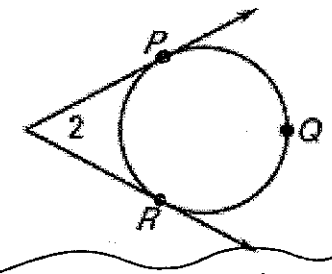
Tangent & Secant:



∠1 intercepts arcs $\overset{\frown}{AC}$ & $\overset{\frown}{BC}$

$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - \overset{\frown}{AC})$

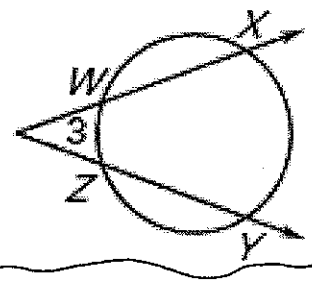
Two Tangents:



∠2 intercepts arcs $\overset{\frown}{PQR}$ + $\overset{\frown}{PR}$

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$

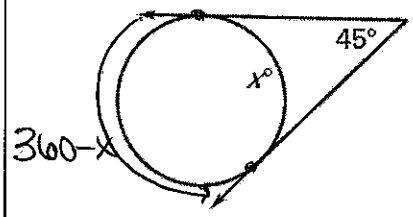
Two Secants:



∠3 intercepts arcs $\overset{\frown}{XY}$ + $\overset{\frown}{WZ}$

$m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - \overset{\frown}{WZ})$

5. $x =$ 135



$45 = \frac{1}{2}(360 - x - x)$

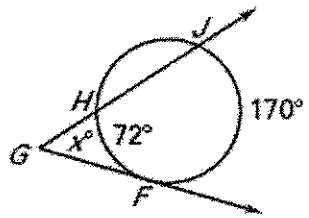
$45 = \frac{1}{2}(360 - 2x)$

$90 = 360 - 2x$

$-270 = -2x$

$135 = x$

6. $x =$ 49

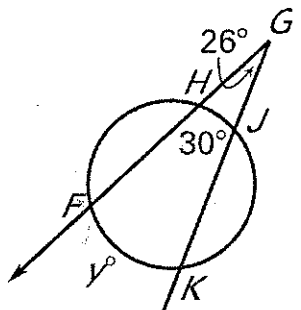


$x = \frac{1}{2}(170 - 72)$

$x = \frac{1}{2}(98)$

$x = 49$

7. $y =$ 82



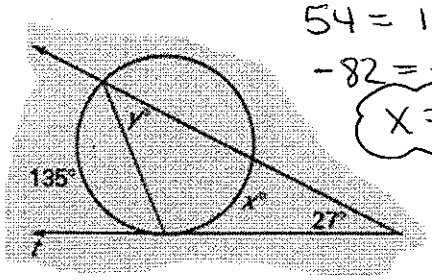
$26 = \frac{1}{2}(y - 30)$

$52 = y - 30$

$82 = y$

Challenge Problems:

8. Find x and y.



$27 = \frac{1}{2}(136 - x)$

$54 = 136 - x$

$-82 = -x$

$x = 82$

$y = \frac{1}{2}x$

$y = \frac{1}{2}(82)$

$y = 41$

9. Find x.

$41 = \frac{1}{2}(124)$

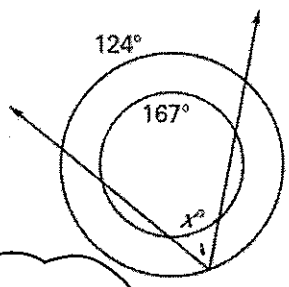
$41 = 62$

$62 = \frac{1}{2}(167 - x)$

$124 = 167 - x$

$-43 = -x$

$43 = x$



$x = 43$