4.1 Discrete Probability

Random Variable - x - represents a numerical value associated with each outcome of a probability experiment

There are two kinds of Random Variables

discrete - finite number, countable

continuous - uncountable, interval

A discrete probability distribution lists each possible value that a random variable can take, along with its probability. It has the following properties:

1. The probability of each value of the discrete random variable is between 0 and 1 inclusive
2. The sum of all the probabilities is 1

X # fish caught	0	1	2	3	4	
frequency	88	72	30	8	2	200
P(x)	88/200	72/200	30/200	8/200	2/200	
P(x)	.44	.36	.15	.04	.01	

μ Expectd

$\boxed{\sum x P(x)}$

$0(.44) + 1(.36) + 2(.15) + 3(.04) + 4(.01) =$

$0 + .36 + .30 + .12 + .04 = \underline{.82}$

Jan 18-10:45 AM

X	P(x)	(x-μ)	(x-μ)²	P(x)(x-μ)²	
0	.44	0-.82	.67	.44(.67) =	.2948
1	.36	1-.82	.03	.03(.36) =	.0108
2	.15	2-.82	1.39	1.39(.15) =	.2085
3	.04	3-.82	4.75	(4.75)(.04)	.19
4	.01	4-.82	10.11	(10.11)(.01)	.1011
				Varianc	.8052

$\mu = .82$

$\sum x P(x)$

σ^2 Variance

$\sqrt{.8052}$

.8973 STANDARD DEVIATION

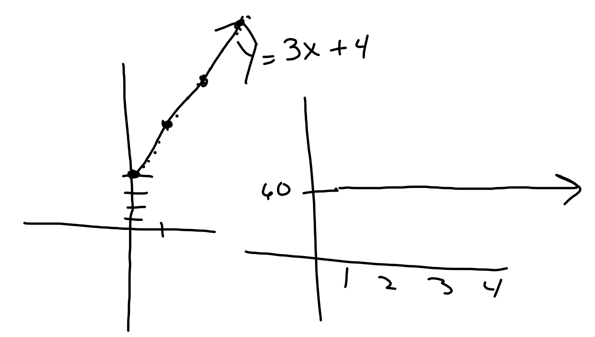
Jan 18-11:21 AM

x	f	P(x)	xP(x)		(x-μ)	(x-μ)²	(x-μ)²P(x)
1	24	.16	1(.16) =	.16			
2	33	.22	2(.22) =	.44			
3	42	.28	3(.28)	.84			
4	30	.20	4(.20)	.80			
5	21	.14	5(.14)	.70			
	T= 150	1.	$\sum xP(x) =$	2.94			

Expected Mean

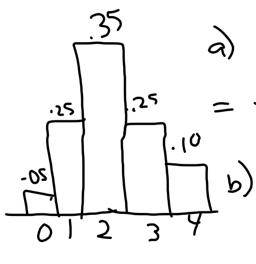
$\sum x P(x)$

Jan 17-8:28 AM



Jan 19-7:58 AM

(21)



a) $= .25 + .10$

$\boxed{.35}$

b) $.05 + .25 + .35 + .25 = .9$

OR $1 - .10 = .9$

Jan 19-8:04 AM

x	f	P(x)	Σ x(P(x))
0	1491	1491 ÷ 2175 = .69	0
1	425	= .20	.20
2	168	= .08	.16
3	48	= .02	.06
4	29	= .01	.04
5	14	= .01	.05
	2175	1.	.50 = μ

Jan 19-8:11 AM

x	$(x-\mu)$	$(x-\mu)^2$	$P(x)(x-\mu)^2$
0	0-.50 =	.25 ×	.69 = .1725
1	1-.50	.25 ×	.20 → .05
2	2-.50	2.25 ×	.08 → .18
3	3-.50	5.06 ×	.02 → .1012
4	4-.50	12.25 ×	.01 → .1225
5	5-.50	20.25 ×	.01 → .2025

$\mu = .5$ $\sigma = \sqrt{.8287}$ σ^2 $\overline{.8287}$

$\boxed{\sigma = .91}$

Jan 19-8:17 AM

Expected Value = mean $xP(x)$

x	f	$P(x)$	$xP(x)$
1	24	24/150 = .16	1(.16) = .16
2	33	33/150 = .22	2(.22) = .44
3	42	42/150 = .28	3(.28) = .84
4	30	30/150 = .20	4(.20) = .80
5	21	21/150 = .14	5(.14) = .70
	150		

$\mu = \Sigma\, xP(x)$ $\underline{2.94}$

Jan 17-11:19 AM

x	$(x-\mu)^2$		$P(x)(x-\mu)^2$
1	$(1-2.94)^2$ =	3.7636	.16(3.7636) = .602176
2	$(2-2.94)^2$ =	.8836	.22(.8836) = .194392
3	$(3-2.94)^2$ =	.0036	.28(.0036) = .001008
4	$(4-2.94)^2$ =	1.1236	.20(1.1236) = .224720
5	$(5-2.94)^2$ =	4.2436	.14(4.2436) = .59104

σ^2 $\sigma = \sqrt{1.6164}$ Variance σ^2 $\underline{1.6164}$

S^2 $S = \sqrt{1.6164}$

1.3

Jan 17-11:43 AM

p196

TOTAL 1500   4 Win   1496 Lose

EACH $2

Prizes

$500

$250

$150

$75

Gains	498	248	148	73	-2
P(x)	1/1500	1/1500	1/1500	1/1500	1496/1500
	.0006	.0006	.0006	.0006	.997

$\mu = \Sigma\, x(P(x))$

$498(.0006) + 248(.0006) + 148(.0006) + 73(.0006) - 2(.997) = \boxed{-\$1.35}$

Jan 17-11:52 AM

2000 Tickets

$5 each

$2000

$1000

$500

$250

$100

Prizes

Gains	1995	995	495	245	95	-5
P(x)	1/2000	1/2000	1/2000	1/2000	1/2000	1995/2000
	.0005	.0005	.0005	.0005	.0005	.998

1995(.0005) + 995(.0005) + 495(.0005) . . .

-$3.08

Jan 17-12:00 PM

p196 Example 7   1 Ticket $2

Total 1500

Losers 1496

$500

$250

$150

$75

Gain	$498	248	148	73	-2
P(x)	1/1500	1/1500	1/1500	1/1500	1496/1500

Expected Value

$\Sigma = 498\left(\frac{1}{1500}\right) + 248\left(\frac{1}{1500}\right) + 148\left(\frac{1}{1500}\right) + 73\left(\frac{1}{1500}\right) + -2\left(\frac{1496}{1500}\right)$

$ -1.35

Jan 17-8:44 AM

X - Gains	1995	995	495	245	95	−5
P(x)	1/2000	1/2000	1/2000	1/2000	1/2000	1995/2000

2000 Tickets

\$5 each

$\boxed{\Sigma x(P(x) = \text{Expected Value}}$

Prizes

2000
1000
500
250
100

$1995\left(\frac{1}{2000}\right) + 995\left(\frac{1}{2000}\right) + 495\left(\frac{1}{2000}\right) +$

$245\left(\frac{1}{2000}\right) + 95\left(\frac{1}{2000}\right) - 5\left(\frac{1995}{2000}\right) = \boxed{\$ -3.08}$

Jan 18-11:55 AM

TOTAL 2000 Tickets

\$5 each

5 winning
1995 Losing

Prizes: 2000
1000
500
250
100

X Gains \$	1995	995	495	245	95	−5
P(x)	$\frac{1}{2000}$ .0005	$\frac{1}{2000}$	$\frac{1}{2000}$	$\frac{1}{2000}$	$\frac{1}{2000}$	$\frac{1995}{2000}$

$\Sigma XP(x) = .9975 + .4975 + .2475 + .1225 + .0475 - 4.9875$

$\boxed{-3.075}$

\$ −3.08

Jan 17-8:51 AM

X − M	(X−μ)	$(X-\mu)^2$	$(x-\mu)^2 P(x)$
1 − 2.94	(−1.94)	3.76	3.76(.16) = .602
2 − 2.94	−.94	.884	.884(.22) = .194
3 − 2.94	.06	.004	.004(.28) = .001
4 − 2.94	1.06	1.124	1.124(.20) = .225
5 − 2.94	2.06	4.244	4.244(.14) = .594

σ^2

s σ

$\sigma^2 = 1.6$

$\sigma = \sqrt{1.6}$

$\boxed{1.3}$

Jan 17-8:34 AM

1500 Tickets

\$2 Each Ticket

Prizes

\$500
\$250
\$150
\$75

x → Gains	498	\$248	148	73	−2
P(x)	1/1500	1/1500	1/1500	1/1500	1496/1500

Jan 18-11:49 AM

35

$0(.02) + 1(.02) + 2(.06) + 3(.06) + 4(.08) + 5(.22)$

$6(.30) + 7(.16) + 8(.08) =$

Expected Value = Mean

$\Sigma \ X(P(x))$

$0 + .02 + .12 + .18 + .32 + 1.1 + 1.8 + 1.12 + .64 =$

a) $\mu = 5.3$

$(X-\mu)^2 P(x)$

$(0-5.3)^2(.02) = .5618$

$(1-5.3)^2(.02) = .3698$

$(2-5.3)^2(.06) = .6534$

$(3-5.3)^2(.06) = .3174$

$(4-5.3)^2(.08) = .1352$

$(5-5.3)^2(.22) = .0198$

$(6-5.3)^2(.30) = .147$

$(7-5.3)^2(.16) = .4624$

$(8-5.3)^2(.08) = .5832$

$\sigma^2 = 3.25$

$\sigma = \sqrt{3.25}$

$\sigma = 1.8$

Jan 19-8:33 AM

Jan 19-8:45 AM