Section 3.3, The Addition Rule

The probability that either event A or B (or both) will happen is:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If $P(A \text{ or } B) = P(A) + P(B)$ (i.e. $P(A \text{ and } B) = 0$), we say that A and B are **mutually exclusive** or **disjoint**. In other words, the events cannot happen at the same time.

Examples
Calculate each of the following probabilities, and state whether the events are mutually exclusive or not:

1. You roll a 5 or a 6 on a standard die.
   These events are mutually exclusive since it isn't possible to roll both a 5 and a 6 at the same time. So, $P(5 \text{ or } 6) = P(5) + P(6) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$.
2. You draw a 4 or a heart from a standard deck of cards.
   These events are not mutually exclusive, since you can draw a card that is both a 4 and a heart (the 4 of hearts). So, $P(4 \text{ or } \heartsuit) = P(4) + P(\heartsuit) - P(4 \text{ and } \heartsuit) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$.
3. You draw a spade or a club from a standard deck of cards.
   These events are mutually exclusive, so $P(\spadesuit \text{ or } \clubsuit) = P(\spadesuit) + P(\clubsuit) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$.
4. The following table shows the Myers-Briggs personality preference and area of study for a random sample of 519 college students.

Myers-Briggs Preference	Arts and Science	Business	Allied Health
IN (introvert, intuitive)	64	15	17
EN (extrovert, intuitive)	82	42	30
IS (introvert, sensing)	68	35	12
ES (extrovert, sensing)	75	42	37

   Find each of the following probabilities for a student chosen at random:

   (a) The student is an intuitive extrovert.
   We will calculate this by adding the probabilities over each of the following majors:

$$P(\text{EN}) = \frac{82}{519} + \frac{42}{519} + \frac{30}{519} = \frac{154}{519} = 0.2967.$$

   (b) The student is either a sensing extroverted business major or an intuitive introverted allied health major.
   These events are mutually exclusive, so the probability that we want is $\frac{42}{519} + \frac{17}{519} = \frac{59}{519} = 0.2237$.

Note: There is a good summary of probability rules on page 160.